Don K. Mak

# MATHEMATICAL TECHNIQUES *in* FINANCIAL MARKET TRADING

World Scientific

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*To*

my parents, whom I am indebted for my upbringing
and education,

and

my wife, whom I am thankful for her loving and care.

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# Preface

I finished writing the book *The Science of Financial Market Trading* in 2002. The book was written for the general public, with intended audience being the traders and investors. A number of computer programs have been included in the book for ease of application. The mathematics was kept to a minimum in the main text while the bulk of the mathematical derivations was placed in the Appendices. However, the book was actually purchased mainly by libraries and bookstores of some of the major universities and research centers around the world. It was further adopted as a textbook for a graduate course in mathematical finance by an American university.

This pleasant surprise may reflect the change in perspectives of university educators toward the trading arena for the last few years. A new discipline called "Financial Engineering" has appeared due to the demand from the financial services industry and economy as a whole. The explosive growth of computer technology and today's global financial transaction have led to a crucial demand of professionals who can quantify, appraise and predict increasingly complex financial issues. Some universities (mostly in the U.S. and Canada) are beginning to offer M.Sc. and even Ph.D. programs in financial engineering. Computing and trading laboratories are set up to simulate real life situations in the financial market. Students learn how to employ mathematical finance modeling skills to make pricing, hedging, trading, and portfolio management decisions. They are groomed for careers in securities trading, risk management, investment banking, etc.

The present book contains much more materials than the previous book. Spectrum analysis is again emphasized for the characterization of technical indicators employed by traders and investors. New indicators are created. Mathematical analysis is applied

to evaluate the trading methodologies practiced by traders to execute a trade. In addition, probability theory is employed to appraise the utility of money management techniques. The book is organized in fourteen chapters.

Chapter 1 describes why the book is written. This book aims to analyze the equipment that professional traders used, and attempt to distinguish the tools from the junk.

Chapter 2 presents the latest development of scientific investigation in the financial market. A new field, called Econophysics, has cropped up. It involves the application of the principles of Physics to the study of financial markets. One of the areas concerns the development of a theoretical model to explain some of the properties of the stochastic dynamics of stock prices. There exist also growing evidences that the market is non-random, as supported by new statistical tests. In any case, market crashes have been considered to be non-random events. What the signatures are before a crash and how a crash can be forecasted will be described.

Chapter 3 analyzes the trending indicators used by traders. The trending indicators are actually low pass filters. The amplitude and phase response of one of the most popular indicators, the exponential moving average, is characterized using spectrum analysis. Other low pass filters, the Butterworth and the sinc functions are also looked into. In addition, an adaptive exponential moving average, whose parameter is a function of frequency, is introduced.

Chapter 4 modified the exponential moving average such that new designs would have less phase or time lag than the original one. It also pointed out that the "Zero-lag" exponential moving average recently designed by a trader does not live up to its claim.

Chapter 5 describes causal wavelet filters, which are actually band-pass filters with a zero phase lag at a certain frequency. The Mexican Hat Wavelet is used as an example. Calculation of the frequency where the zero phase lag occurs is shown. Furthermore, it is demonstrated how a series of causal wavelet filters with different frequency ranges can be constructed. This tool will allow the traders to monitor the long-term, mid-term and short-term market movements.

Chapter 6 introduces a trigonometric approach to find out the instantaneous frequency of a time series using four or five data points. The wave velocity and acceleration are then deduced. The method is then applied to theoretical data as well as real financial data.

Chapter 7 explains the relationship between the real and imaginary part of the frequency response function of a causal system, $H(\omega)$. Given only the phase of a system, a method is implemented to deduce $H(\omega)$. Several examples are given. The phase or time response of a system or indicator is important for a trader tracking the market movements. The method would allow them to predetermine the phase, and work backward to find out what the system is like.

Chapter 8 depicts several newly created causal high-pass filters. The filters are compared to the conventional momentum indicator currently popular with traders. Much less phase lags are achieved with the new filters.

Chapter 9 describes in detail the advantages and limitations of a new technique called skipped convolution. Skipped convolution, applied to any indicator, can alert traders of a trading opportunity earlier. However, it also generates more noise. A skipped exponential moving average would be used as an example. Furthermore, the relationship between skipped convolution and downsampled signal is illustrated.

Chapter 10 analyzes and dissects some of the popular trading tactics employed by traders, in order to differentiate the truths from the myths. It explains the meaning behind divergence of momentum (or velocity) from price. It unravels the significance of the MACD (Moving Average Convergence-Divergence) line and MACD-Histogram, but downplays the importance of the MACD-Histogram divergence.

Before putting up a trade, traders would look at charts of different timeframes to track the long-term and short-term movements of the market. The advantages and disadvantages of a long-term timeframe are pointed out in Chapter 11. This chapter also discusses how a trading plan should be put together. The popular Triple Screen Trading System is used as one of the examples.

The market is assumed to be random in Chapters 12 and 13. This modeling is good as a first approximation, and renders the application of probability theory to money management techniques practiced by traders. Chapter 12 discusses the profitability of the market at any moment in time. Chapter 13 derives and computes how traders can optimize their gain by moving the stop-loss.

The final chapter, Chapter 14, discusses the reality of financial market trading. It takes years of hard work and training to be a successful trader. In addition, the trader needs to update himself of current technology and methodology in order to keep ahead of the game.

Most of the mathematical derivations and several computer programs are listed in the Appendices.

Writing this book takes many hours of my time away from the company of my two adorable children, Angela and Anthony; and my beautiful wife, Margaret, whom I am very thankful for.

*D. K. Mak*
2005

# Contents

## Chapter 1

# Introduction

Scientific theories quite often go through three stages of development: – (1) Absurdity – the idea or theory sounds so absurd that one wonders why someone would have suggested it, (2) Familiarity – there appears to be growing evidence to support the hypothesis, and people begin to familiarize themselves with the concept, and (3) Inevitability – the theory becomes so obvious in hindsight that people would think why it was not recognized earlier and why it has taken so long for the community to come to accept it.

Is the financial market not random? Fifty years ago, the academia would think it was ridiculous to say that the market was non-random. Since then, there have appeared journal papers challenging the random walk theory. At the moment, some academics would conclude that the market is non-random (see details in Chapter 2). However, the debate is still on, and there could be many years before the final verdict is in.

During all these time, the market traders could not care less what the academics think. They swear, by their own observation and experience, that the market is not random. Some even claim even if it were random, with good money management, they can still make a profit from the market. They facilitate their own methods to trade. Some do consistently make money from the market year after year. They design indicators to forecast which way the market is heading. And they devise trading systems to enter and exit the market. However, no trader seems to care to analyze their indicators and methodologies mathematically, nor do they try to characterize them. Their tools range from the very useful to complete garbage.

This scenario is somewhat similar to alternative medicine thirty years ago. Then, alternative medicine was unconventional, unproven, and unorthodox, and was ignored by the mainstay medical researchers. However, some of the alternate approaches do represent many years of experience of the practitioners by trial and error, and can contain some truths. They may even depict innovative means to problems conventional medicine has no cure. But, then, of course, some of the alternative medicine is eccentric and harmful. It was fortunate that medical researchers did finally take a serious note at these alternative therapies, and apply scientific methods to study them. It would be up to them to differentiate the grass from the weeds.

The tools employed by the market traders have a similar script. Some professional traders, by trial and errors, pick certain indicators as their arsenals, and make consistent profits from the market, even though they do not exactly understand the properties of their accouterments. Other traders advertise their indicators, and black box methodologies, and claim they can perform miracles. Believers wind up losing their shirts in the market.

It is the purpose of this book to analyze their tools mathematically, and display their characteristics. Spectrum analysis is emphasized. Some of the ideas have been presented earlier [Mak 2003]. We will expand on those ideas. We will point out why some of the traders' techniques work, and why some do not. In addition, we will also look at how a good trading plan can be put together, and how, according to probability theory, some of the money management techniques employed by traders do make profitable sense. Furthermore, we will invent some new indicators, which have less time or phase lag than the ones currently used by traders. These would allow them to pick up market signals earlier. We hope that this presentation will be useful to the trading community.

Chapter 2

# Scientific Review of the Financial Market

How the financial market has been modeled in different endeavors has been described by Mak [2003]. From all perspectives, it seems as if it would be best modeled as a complex phenomenon. A complex system contains a number of agents who are intelligent and adaptive. The agents make decisions on the basis of certain rules. They can modify old rules or create new rules as new information arises. They know at most what a few other agents are doing. They then decide what to do next based upon this limited information [Waldrop 1992, Casti 1995, Johnson et al 2003]. Scientists and mathematicians have been trying to draw some conclusions from the complex financial system. Some of their recent attempts are described below.

## 2.1 Econophysics

Over the past two decades, a growing number of physicists has become involved in the analysis of the financial markets and economic systems. Using tools developed in statistical mechanics, they were able to contribute to the modelling of the dynamics of the economy in a practical fashion. A new field, known as econophysics, has thus emerged [Mantegna and Stanley 2000]. The field benefits from the large database of economic transactions already recorded. Several findings are described below.

### 2.1.1 *Log-Normal Distribution of Stock Market Data*

In 1900, Bachelier wrote that price change in the stock market followed a one-dimensional Brownian motion, which has a normal (Gaussian)